Lemonade for a Party
Alberto and Kisha are in charge of bringing lemonade to their class party. They each made a pitcher of lemonade. Alberto's lemonade has three lemons and 2 cups of water. Kisha's lemonade has 4 lemons and 3 cups of water. They only want to take the most lemony lemonade to the party. Who's lemonade is the most lemony?
Once you decide which is the most lemony, prove that you are have made the right choice.

Prior Student Knowledge:
Students do not need prior knowledge of ratio to participate in this lesson. It is helpful, but not essential that students know the fraction 1/2.

Content Standards:
N.S. 1.2- Interpret and use ratios in different contexts to show the relative sizes of 2 quantities, using appropriate notations.
M.R. 1.2- Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
M.R. 2.4- Use a variety of methods such as words, numbers, or symbols, charts, graphs, tables, diagrams and models to explain mathematical reasoning.
M.R. 3.3- Develop generalizations of the results obtained and the strategies used and apply them in new problem situations.

Learning Objectives:
Students will understand that a ratio can describe one part in relationship to another part.
Students will explore the relationship between two parts using their own generated methods.
Students will compare ratios using "fair share."
Students will make conjectures and explore ways of testing their conjectures using counter-examples.
Students will learn that for a strategy to be useful mathematically, it must work in all cases.

Assessment Strategy:
Students' performance on comparing the "lemony-ness" of two pitchers of lemonade will demonstrate their understanding of ratio as a part to part relationship.
Students written conjectures on which is more lemony will reveal their thinking about comparing two quantities in relationship to one another.
Final problem, will show teachers their student's thinking on proportion in preparation for the next lesson.

Description of Lesson Flow:
Students will be presented with an introductory problem: two pitchers of lemonade have an identical number of lemons- which pitcher is more lemony? Students will share their ideas about this introductory problem. Next, a different amount of water will be added to each of the pitchers of lemonade and students are again asked which is more lemony. Students will discuss their answers and their rationales. Next, students will be presented with the Lemonade for a Party problem. In pairs, they will write down their solutions and rationales. The teacher will circulate around the room and take note of different solutions and rationales that students give. Teacher will lead a

discussion on the Lemonade problem taking note of the different rationales for the given solutions. Teacher will facilitate a discussion where students prove/dis-prove their conjectures about which pitcher is more lemony. Teacher will lead a lesson on "fair share" demonstrating the distribution of lemons into the cups of water and defining ratio as the relationship between two quantities. Students will re-visit the Lemonade problem making use of what they have learned about "fair share." Lesson will conclude with an additional problem where students compare the lemony-ness of two pitchers of lemonade.

Placement in Curriculum:
Sixth grade is the first introduction that students have to ratio. As well, it is not included in the 7^{th} and 8^{th} grade standards. However, understanding how to make part to part comparisons is a very important part of success in middle school mathematics (need reference.)

Materials:
Lemonade handout; 5 X 7 cards or post-its for name plates; markers and large chart paper for writing conjectures

Time	Activity	Anticipated Student Responses	Teacher Response	Annotation
5 min	Students will be presented with an introductory problem: Two pitchers of lemonade have an identical number of lemons- which pitcher is more lemony? Students will share their ideas about this introductory problem. Next, a different amount of water will be added to each of the pitchers of lemonade and students are again asked which is more lemony. Students will discuss their answers and their rationales.	-Students will not know how to answer the first question thinking that the answer is obvious. -Students will say that the two pitchers taste the same because they have the same # or lemons. - Students will say that the problem cannot be answered because we do not know how much water is in the pitcher.	-Accept each answer as equally valid. (Do not give more credence to one answer over the other.) -Encourage students to voice not only their answer but also the rationale for their answers.	
10 min	Students will be presented with the Lemonade for a Party problem. In pairs, they will write down their solutions and rationales. The teacher will circulate around the room and take note of different solutions and rationales that students give.	-Students may ask the teacher for assistance or not know what they are supposed to do. -Students may not be used to proving their solutions or writing explanations.	-Circulate throughout the classroom and take note of the student conversations and solutions.	

Time	Activity	Anticipated Student Responses	Teacher Response	Annotation
5 min	-Ask students to share their conjectures about who has the more lemony lemonade. - As students give their conjectures, write them up on the chart paper	1. Students will say that Alberto's lemonade is more lemony because it has less water. -Students will say that Kisha's lemonade is more lemony because it has more lemons. 2. Students will say that Kisha's is more lemony because after a fair share, each cup has 1 2/3 lemons while Alberto's cups each have 1 ½ lemons. 3. Students will say that Kisha's is more lemony because there are 3 lemons left over and only one cup of water.	-Encourage students to share their conjectures. If they do not, select specific students to share based on the solutions that you noted during student discussions. -Make sure that you have each of the anticipated responses on the board.	

Time	Activity	Anticipated Student Responses	Teacher Response	Annotation
10 min	-Ask students how we can tell which of the solutions is correct. -Present the idea of conjecture- for a conjecture to be true, it must be true in all situations -Explain to students that we are going to try to prove which, if any, of our conjectures is true. -Present a counter example to conjecture A asking- "Can we find an example where there is less water and the lemonade is still less lemony?" -Present a counter example to conjecture B asking- "Can we find an example where there are more lemons and the lemonade is still less lemony?"	-Students may have difficulty letting go of their conjecture- even after a counter example which disproves theirs is presented. 1. Students will say that Alberto's lemonade is more lemony because it has less water. -Students will say that Kisha's lemonade is more lemony because it has more lemons. 2. Students will say that Kisha's is more lemony because after a fair share, each cup has 1 2/3 lemons while Alberto's cups each have 1 ½ lemons. 3. Students will say that Kisha's is more lemony because there are 3 lemons left over and only one cup of water.	-Throughout this part of the lesson, continually check for student understanding. The point is not to tell students that their conjectures are erroneous, but to help them consider an instance when their conjecture is not true. 1. Disprove with A-10 lemons and 5 cups of water and B- 1 lemon and 4 cups of water 2. Disprove with A-2 lemons and 2 cups of water and B- 3 lemons and 6 cups of water. 3. Use as a segue into "fair-share." i.e. Okay, but we cannot have any lemons left over, so what do we do with the extra lemons?	
10 min	- Show students that the extra lemons should be divided amongst the cups of water evenly. (Base this upon the 3rd conjecture above). -As a whole class, return to and solve the main problem, and each of the other 2 conjectures- using fair share.	-Students with limited knowledge of fraction sizes may have difficulty	-Remind students that for a conjecture to be true- it must work in all instances.	

Time	Activity	Anticipated Student Responses	Teacher Response	Annotation
15 min	-Students will be presented with similar problems (B-D) and asked to solve them individually.	- The majority of students in the class should be able to answer the problem using the method of fair share. -There may still be a number of students who answer based upon the first “dis-proven” conjectures. It is important to point out these mis-conceptions and re-visit the dis-proven conjectures.		